Math 72 4.5 - 2nd Applications of Linear Systems

Objectives

1) Set up and solve application problems that result in linear systems

- mixture problems
  - % solutions
  - money
  - other
- uniform motion
- direct translation
- geometry
- break-even

Key Concept:

- Every problem requires 2 or 3 variables
- The number of equations needed = number of variables

In each example, the resulting system can be solved using multiple methods. Only one method is demonstrated.

Math 72   4.5 - 2nd Applications of Linear Systems.

Key concept: # equations = # variables

① Terry can row 10.6 km in 3 hours downstream and 6.8 km upstream in 2 hours. Find how fast Terry can row in still water and speed of current.

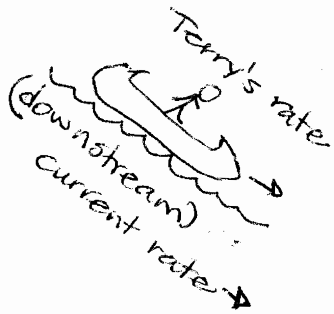
Explanation: Still Water, Downstream, Upstream.

If Terry goes anywhere, it's because Terry rows.

Terry's rate in still water $= x$

rate of the water $= 0$.

combined rate $= x + 0 = x$.

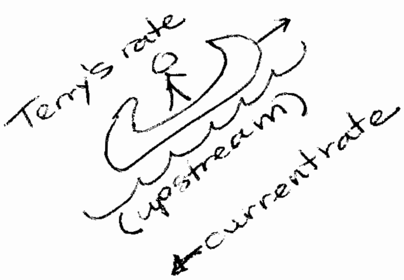


Terry still has the same capacity to row.

Terry's rate in still water $= x$

rate of the water $= y$ (helping)

combined rate $= x + y$



Terry still has the same capacity to row.

Terry's rate in still water $= x$

rate of water $= y$ (against)

combined rate $x - y$.

Note: Terry won't actually <u>go</u> upstream unless $x > y$!

Uniform motion formula

$$D = R \cdot T$$

	D	R	T
down	10.6	$x+y$	3
up	6.8	$x-y$	2

Multiply R·T

$$\begin{cases} 10.6 = 3(x+y) & \text{Ⓐ} \\ 6.8 = 2(x-y) & \text{Ⓑ} \end{cases}$$

2 equations, 2 unknown variables

Solve the system by substitution or elimination.
Eliminate y:

Divide Ⓐ by 3 $\qquad \frac{10.6}{3} = x + y$

Divide Ⓑ by 2 $\qquad \frac{6.8}{2} = x - y$

$$\left.\begin{aligned} \frac{53}{15} &= x + y \\ 3.4 &= x - y \end{aligned}\right\} \quad \text{elimination}$$

$$\frac{104}{15} = 2x$$

$$\frac{52}{15} = x$$

$$3.4 = \frac{52}{15} - y$$

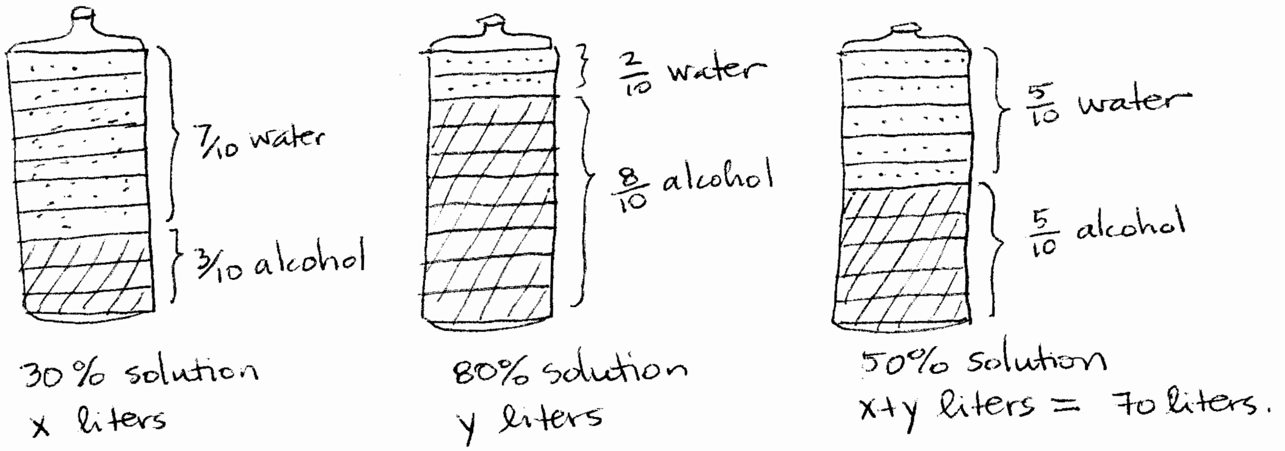
$$y = \frac{52}{15} - 3.4$$

$y = \boxed{\frac{1}{15}\text{ kph} = 0.0\overline{6}\text{ kph} \text{ speed of current}}$

$x = \boxed{\frac{52}{15}\text{ kph} = 3.4\overline{6}\text{ kph} \text{ speed in still water}}$

(2) Lynn Pike, a pharmacist, needs 70 L of a 50% alcohol solution. She has 30% and 80% solutions available. How many liters of each should she mix to obtain 70 L of 50% solution?

Explanation: 30%, 80% and 50% solutions.



Equations:

volume of liquid: x liters + y liters = 70 liters

liters of alcohol: 30% of x + 80% of y = 50% of 70 liters

$$.3x + .8y = .5(70)$$

$$.3x + .8y = 35$$

2 eqns, 2 unknown variables:

$$\left.\begin{aligned} x + y &= 70 \\ .3x + .8y &= 35 \end{aligned}\right\} \quad \begin{matrix} A \\ B \end{matrix}$$

Solve system by substitution or elimination.

Substitute for y

Solve A for $y = 70 - x$

Subst into B $.3x + .8(70 - x) = 35$

$$.3x + 56 - .8x = 35$$

$$56 - .5x = 35$$

$$-.5x = -21$$

$$x = \frac{-21}{-.5} = \boxed{42 \text{ liters of } 30\%}$$

$$y = 70 - 42 = \boxed{28 \text{ liters of } 80\%}$$

(3) Rabbits in a laboratory eat a strict diet of exactly 30 g protein, 15 g fat, and 24 g carbohydrates. The scientist has three brands of rabbit food which provide the following quantities of each nutrient in each scoop.

	protein	fat	carbs
Brand A	$\frac{4\text{ g protein}}{\text{scoop}}$	$\frac{6\text{ g fat}}{\text{scoop}}$	$\frac{3\text{ g carbs}}{\text{scoop}}$
Brand B	6 g	1 g	2 g
Brand C	4 g	1 g	12 g

How many scoops of each brand should be mixed to meet the dietary requirements? Round to the nearest tenth, if necessary.

Variables to answer the question:

a = # scoops of brand A

b = # scoops of brand B

c = # scoops of brand C.

Explanation: Consider only protein, and only brand A.

For example: 2 scoops of A gives $\frac{4\text{ g}}{\cancel{\text{scoop}}} \times 2\ \cancel{\text{scoops}} = 8$ grams

a scoops of A gives $\frac{4\text{ g}}{\cancel{\text{scoop}}} \times a\ \cancel{\text{scoops}} = 4a$ grams.

$$\begin{cases} \text{Total protein: } \underbrace{4a}_{\text{from A}} + \underbrace{6b}_{\text{from B}} + \underbrace{4c}_{\text{from C}} = \underbrace{30\text{ g}}_{\text{total.}} \text{ protein.} & (1) \\ \text{Total fat: } 6a + 1b + 1c = 15\text{ g fat.} & (2) \\ \text{Total carbs: } 3a + 2b + 12c = 24\text{ g carbs} & (3) \end{cases}$$

3 equations, 3 unknown variables.

Solve by eliminating the same variable twice OR substituting for one variable in two equations.

Substitution eq'n (2): $b = 15 - 6a - c$

Substitute into eq'n (1)

$$4a + 6(15-6a-c) + 4c = 30$$
$$4a + 90 - 36a - 6c + 4c = 30$$
$$-32a - 2c = -60$$
$$32a + 2c = 60$$
$$\underline{16a + c = 30} \quad (4)$$

Substitute into eq'n (3)

$$3a + 2(15-6a-c) + 12c = 24$$
$$3a + 30 - 12a - 2c + 12c = 24$$
$$\underline{-9a + 10c = -6} \quad (5)$$

2 equations, 2 unknown variables:

$$\begin{cases} 16a + c = 30 & (4) \\ -9a + 10c = -6 & (5) \end{cases}$$

Solve (4) for c: $c = 30 - 16a$

Subst into (5): $-9a + 10(30-16a) = -6$

$$-9a + 300 - 160a = -6$$
$$-169a = -306$$
$$a = \frac{306}{169}$$

MATH
1. ▸frac

* CAUTION *
Use exact answers to solve for b and c!!
Then round at the very end only.

Subst back: $c = 30 - 16\left(\frac{306}{169}\right) = \frac{174}{169}$

Subst back: $b = 15 - 6\left(\frac{306}{169}\right) - \frac{174}{169} = \frac{525}{169}$

Round as specified

$a \approx 1.81$
$b \approx 1.02$
$c \approx 3.10$

$$\boxed{\begin{array}{l} \text{1.8 scoops Brand A} \\ \text{1.0 scoops Brand B} \\ \text{3.1 scoops Brand C} \end{array}}$$

(4) A drafting student bought three templates and a pencil for \$6.45, then went back and bought two pads of paper and four pencils for \$7.50. If the price of a pad of paper is three times the price of a pencil, find the prices of each type of item.

x = Template price
y = pencil price
z = paper price

$\frac{\$x}{\text{template}} = \$x$ per template

Direct translation of first phrase

3 times price per template + price per pencil = 6.45

$$3x + y = 6.45$$

Second phrase

2 times price per pad of paper + 4 times price per pencil = 7.50

$$2z + 4y = 7.50$$

Last phrase

price of pad of paper = 3 times price per pencil

$$z = 3y$$

3 equations, 3 unknown variables

$$\begin{cases} 3x + y = 6.45 & \text{A} \\ 4y + 2z = 7.50 & \text{B} \\ z = 3y & \text{C} \end{cases}$$

subst C into B:

$$4y + 2(3y) = 7.50$$
$$4y + 6y = 7.50$$
$$10y = 7.50$$
$$y = 0.75$$

subst into C $\quad z = 3(0.75) = 2.25$

subst into A $\quad 3x + 0.75 = 6.45$

$$3x = 5.7$$
$$x = 1.9$$

$\boxed{\begin{array}{l} x = \$1.90 \text{ per template} \\ y = \$0.75 \text{ per pencil} \\ z = \$2.25 \text{ per pad of paper} \end{array}}$

(5) The measure of the largest angle of a triangle is 80° more than the measure of the smallest angle, and the measure of the remaining angle is 10° more than the measure of the smallest angle. Find the measure of each angle.

Concept / Key Words: Angles of a Triangle
Sum of three angles is 180°.

Unknown variables: $x, y, z \Rightarrow 3$ angles.

let $x =$ smallest

1st phrase: largest $= 80 +$ smallest

2nd phrase: remaining $= 10 +$ smallest

concept: smallest + largest + remaining $= 180$

$x \qquad y \qquad z$

1st phrase:
2nd phrase:
concept:

$$\begin{cases} y = 80 + x & \text{(A)} \\ z = 10 + x & \text{(B)} \\ x + y + z = 180 & \text{(C)} \end{cases}$$

Substitute (A) and (B) into (C):

$$x + (80 + x) + (10 + x) = 180$$

$$3x + 90 = 180$$

$$3x = 90$$

$$\boxed{x = 30^\circ \text{ smallest}}$$

Subst back into (A)

$$y = 80 + 30 = \boxed{110^\circ = y \text{ largest}}$$

Subst back into (B)

$$z = 10 + 30 = \boxed{40^\circ = z \text{ remaining}}$$

check: $30 + 110 + 40 = 180$ ✓

(6) A manufacturing company recently purchased \$3000 worth of new equipment to make personalized stationery. The cost of producing a package of personalized stationery is \$3.00 and it is sold for \$5.50. Find the number of packages that must be sold to break even.

Concept: Break-even Costs = Revenues

costs = fixed costs + Variable costs.

$\left(\text{paid once}\right) + \left(\frac{\text{price}}{\text{Item}}\right) * \left(\#\text{ items}\right)$

$x = \#$ packages

costs $= 3000 + 3x$

revenues $= 5.5x$

$$3000 + 3x = 5.5x$$

$$3000 = 2.5x$$

$$\frac{3000}{2.5} = x$$

$$x = \boxed{1200 \text{ packages}}$$

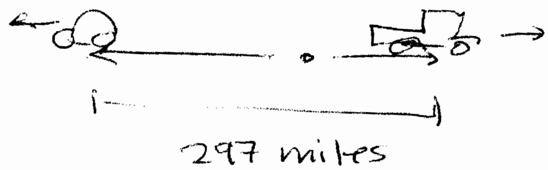
Why is this in the linear system section?
You can introduce another variable y = money

$$\begin{array}{ll} \text{costs:} & \left\{ \begin{array}{l} y = 3000 + 3x \\ y = 5.5x \end{array} \right. \\ \text{revenues:} & \end{array}$$

The y-value $5.5(1200) = \$6600$ is the amount of money paid out when 1200 packages are made and \$6600 is the amount of money received when 1200 packages are sold.

(7) Two cars leave Indianapolis, one traveling east and the other west. After 3 hours they are 297 miles apart. If one car is traveling 5 mph faster than the other, what is the speed of each?

Key words: miles, hours, mph, speed, east/west (opposites)
Concept: uniform motion
Formula: $D = R \cdot T$



D	= R	· T
$3x$	x	3
$3y$	y	3

total 297

y could be x+5
$y = x+5$

$$\begin{cases} 3x + 3y = 297 \\ y = x+5 \end{cases}$$

$$3x + 3(x+5) = 297$$
$$3x + 3x + 15 = 297$$
$$6x + 15 = 297$$
$$6x = 282$$
$$x = \boxed{47 \text{ mph}}$$
$$y = 47 + 5 = \boxed{52 \text{ mph}}$$

⑧ A pharmacist needs 500 ml of a 20% phenobarbitol solution but has only 5% and 25% phenobarbitol solutions. Find how many milliliters of each are needed to get the desired solution.

x = # ml 5%

y = # ml 25%

volume mixed $\quad x + y = 500$

amount of pure phenobarbitol:

$$(5\% \text{ of } x) + (25\% \text{ of } y) = (20\% \text{ of } 500)$$

$$.05x + .25y = 100$$

system of equations:

$$\begin{cases} x + y = 500 & \text{(A)} \\ .05x + .25y = 100 & \text{(B)} \end{cases}$$

Substitution method; solve (A) for y

$$y = 500 - x$$

subst into (B)

$$.05x + .25(500 - x) = 100$$

$$.05x + 125 - .25x = 100$$

$$125 - .2x = 100$$

$$-.2x = -25$$

$$x = \frac{-25}{-.2} = \boxed{125 \text{ ml } 5\%}$$

subst back $\quad y = 500 - 125$

$$= \boxed{375 \text{ ml } 25\%}$$